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**PURSUIT–EVASION PROBLEMS UNDER NONLINEAR  
INCREASE OF THE PURSUER’S RESOURCE****B. T. Samatov, B. I. Juraev**

In the paper, we investigate pursuit-evasion problems in a simple motion differential game with two players, termed a pursuer and an evader. We put different kinds of non-stationary integral constraints, which restrict the energy consumption rate of the players. On the other hand, it is assumed that at each time the players have some additional amount of control resource. The integral constraint on the control of the pursuer is given under certain conditions, which include a non-stationary integral constraint. Firstly, the reachable set of each player is determined. We put forward the parallel approach strategy, which is known as a  $\Pi$ -strategy, for the pursuer, and as a result, we get necessary and sufficient conditions of capture. To solve the evasion problem, a specific admissible strategy is provided for the evader and a sufficient condition is obtained. Furthermore, in the pursuit problem, an optimal capture time is found through the strategy of the evader. In order to illustrate the obtained results, several examples are given, where guaranteed capture times are proposed for the pursuit problems and lower bounds for the distances between the players are obtained for the evasion problem. This work extends the results and methods from the works of R.Isaacs, L.A.Petrosjan, N.N.Krasovskii, A.A.Chikrii, A.A.Azamov, and other authors.

Keywords: pursuit-evasion differential games, simple motion, non-stationary integral constraint, pursuer, evader, strategy, guaranteed capture time.

**REFERENCES**

1. Azamov A.A. On the quality problem for simple pursuit games with constraint. *Serdica Math. J.*, 1986, vol. 12, no. 1, pp. 38–43.
2. Azamov A.A., Kuchkarov A.Sh., Samatov B.T. The relation between problems of pursuit, controllability and stability in the large in linear systems with different types of constraints. *J. Appl. Math. Mech.*, 2007, vol. 71, no. 2, pp. 229–233. doi: 10.1016/j.jappmathmech.2007.06.006.
3. Azamov A.A., Samatov B.T. The  $\Pi$ -Strategy: Analogies and applications. In: *The Fourth Internat. Conf. Game Theory and Management*, St. Petersburg, 2011, vol. 4, pp. 33–46.
4. Azimov A.Ya. Linear differential pursuit games with integral restrictions on the control. *Differential Equations*, 1975, vol. 11, no. 10, pp. 1283–1289.
5. Berkovitz L.D. Differential game of generalized pursuit and evasion. *SIAM J. Contr.* 1986, vol. 24, no. 3, pp. 361–373. doi: 10.1137/0324021.
6. Chikrii A.A. *Conflict-controlled processes*. Dordrecht: Kluwer Academic Publishers, 1997, 404 p. doi: 10.1007/978-94-017-1135-7.
7. Chikrii A.A., Belousov A.A. On linear differential games with integral constraints. *Proc. Steklov Inst. Math. (Suppl.)*, 2010, vol. 269, suppl. 1, pp. S69–S80. doi: 10.1134/S0081543810060076.
8. Dar’in A.N., Kurzhanskii A.B. Control under indeterminacy and double constraints. *Differential Equations*, 2003, vol. 39, no. 11, pp. 1554–1567. doi: 10.1023/B:DIEQ.0000019347.24930.a3.
9. Fleming W.H. The convergence problem for differential games, II. In: *Advances in Game Theory (2nd ed.)*, *Annals of Math.*, 1964, Vol. 52, pp. 195–210. doi: 10.1515/9781400882014.
10. Friedman A. *Differential Games*. Ser. Pure Appl. Math., vol. 25, NY: John Wiley and Sons Inc, 1971, 350 p. ISBN: 0471280496.
11. Gomoyunov M.I. Linear-convex guarantee optimization problems with control delay. *Izv. IMI UdGU*, 2015, vol. 45, no. 1, pp. 37–105 (in Russian).
12. Guseinov Kh.G., Nazlipinar A.S. Attainable sets of the control system with limited resources. *Trudy Inst. Mat. Mekh. UrO RAN*, 2010, vol. 16, no. 5, pp. 261–268 (in Russian).

13. A. Huseyin, N. Huseyin, Kh. Guseinov. Approximation of sections of the set of trajectories for a control system with bounded control resources. *Trudy Inst. Mat. Mekh. UrO RAN*, 2017, vol. 23, no. 1, pp. 116–127 (in Russian). doi: 10.21538/0134-4889-2017-23-1-116-127.
14. Ibragimov G.I., Ferrara M., Kuchkarov A.Sh., Pansera B.A. Simple motion evasion differential game of many pursuers and evaders with integral constraints. *Dyn. Games Appl.*, 2018, vol. 8, no. 2, pp. 352–378. doi: 10.1007/s13235-017-0226-6.
15. Ibragimov G.I., Kuchkarov A.Sh. Fixed duration pursuit-evasion differential game with integral constraints. *J. Phys.: Conf. Ser.*, 2013, vol. 435, art. no. 012017, 12 p. doi: 10.1088/1742-6596/435/1/012017.
16. Isaacs R. *Differential games*. NY: John Wiley and Sons, 1965, 385 p. ISBN: 0471428604.
17. Kornev D.V., Lukoyanov N.Yu. On a minimax control problem for a positional functional under geometric and integral constraints on control actions. *Proc. Steklov Inst. Math.*, 2016, vol. 293, no. 1, pp. 85–100. doi: 10.1134/S0081543816050096.
18. Krasovskii N.N. *Teoriya upravleniya dvizheniem* [Theory of motion control]. Moscow: Nauka Publ., 1968, 476 p.
19. Krasovskii N.N., Repin Yu.M., Tretyakov V.E. Some game situations in the theory of control systems. *Izv. Akad. Nauk SSSR, Ser. Tekhn. Kibernet.*, 1965, no. 4, pp. 3–23 (in Russian).
20. Mamadaliev N.O. Linear differential pursuit games with integral constraints in the presence of delay. *Math. Notes*, 2012, vol. 91, no. 5, pp. 704–713. doi: 10.1134/S0001434612050124.
21. Mezencev A.V. Sufficient escape conditions for linear games with integral constraints. *Dokl. Akad. Nauk SSSR*, 1974, vol. 218, no. 3, pp. 1021–1023.
22. Nikolskii M.S. The direct method in linear differential games with integral constraints. *Controlled systems, IM, IK, SO AN SSSR*, 1969, no. 2, pp. 49–59.
23. Petrosjan L.A. *Differential games of pursuit*. Series on Optimization, vol. 2, Singapore: World Scientific Publishing, 1993, 326 p. ISBN: 9810209797.
24. Pontryagin L.S. *Izbrannye trudy* [Selected Works]. Moscow: MAKS Press, 2004, 551 p.
25. Pshenichnii B.N., Onopchuk Yu.N. Linear differential games with integral constraints. *Izv. Akad. Nauk SSSR, Ser. Tekhn. Kibernet.*, 1968, no. 1, pp. 13–22.
26. Samatov B.T. The pursuit-evasion problem under integral-geometric constraints on pursuer controls. *Autom. Remote Control*, 2013, vol. 74, no. 7, pp. 1072–1081. doi: 10.1134/S0005117913070023.
27. Samatov B.T. Problems of group pursuit with integral constraints on controls of the players I. *Cybernetics and Systems Analysis*, 2013, vol. 49, no. 5, pp. 756–767. doi: 10.1007/s10559-013-9563-7.
28. Samatov B.T., Akbarov A.Kh., Zhuraev B.I. Pursuit-evasion differential games with Gr-constraints on controls. *Izv. IMI UdGU*, 2022, vol. 59, pp. 67–84. doi: 10.35634/2226-3594-2022-59-06.
29. Samatov B.T., Jurayev B.I. The II-strategy in differential game with GrG-constraints on controls. *Bull. Inst. Math.*, 2022, vol. 5, no. 1, pp. 6–13. doi:
30. Samatov B.T., Horilov M.A., Akbarov A.Kh. Differential games with the non-stationary integral constraints on controls. *Bull. Inst. Math.*, 2021, vol. 4, no. 4, pp. 39–46.
31. Satimov N.Yu. *Metody resheniya zadachi presledovaniya v teorii differentsial'nykh igr* [Methods to solve pursuit problems in differential-game theory]. Tashkent: NUUz Press, 2003, 58 p.
32. Satimov N.Yu., Rikhsiev B.B., Khamdamov A.A. On a pursuit problem for  $n$ -person linear differential and discrete games with integral constraints. *Sb. Math.*, 1983, vol. 46, no. 4, pp. 459–471. doi: 10.1070/SM1983v046n04ABEH002946.
33. Subbotin A.I., Chentsov A.G. *Optimizatsiya garantii v zadachakh upravleniya* [Optimization of guaranteed result in control problems]. Moscow: Nauka Publ., 1981, 288 p.
34. Subbotin A.I., Ushakov V.N. Alternative for an encounter-evasion differential game with integral constraints on the players controls. *J. Appl. Math. Mech.*, 1974, vol. 39, no. 3, pp. 367–375. doi: 10.1016/0021-8928(75)90001-5.
35. Tukhtasinov M.  $\varepsilon$ -Positional strategy in the second method of differential games of pursuit. In: *Differential Equations and Dynamical Systems*, Ser. Springer Proceedings in Mathematics & Statistics, vol. 268, Cham: Springer, 2018, pp. 169–182. doi: 10.1007/978-3-030-01476-6\_13.

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