ON THE PROPERTIES OF THE SET OF TRAJECTORIES OF NONLINEAR CONTROL SYSTEMS WITH INTEGRAL CONSTRAINTS ON THE CONTROL FUNCTIONS

N. Huseyin, A. Huseyin, Kh. G. Guseinov

The control systems described by nonlinear differential equations and integral constraints on the control functions are studied. Admissible control functions are chosen from a closed ball of the space L_p, $p \in (1, \infty]$, with radius r and centered at the origin. It is proved that the set of trajectories of the system is continuous at $p = \infty$ with respect to the Hausdorff pseudometric. It is shown that every trajectory is robust with respect to the fast and full consumption of the remaining control resource which implies that to achieve the desired result, it is advisable to spend the available control resource in small portions. This allows to prove that every trajectory can be approximated by the trajectory, generated by full consumption of the control resource.

Keywords: nonlinear control system, set of trajectories, integral constraint, geometric constraint, Hausdorff continuity, robustness.

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Nesir Huseyin, Associate Professor, Sivas Cumhuriyet University, Faculty of Education, 58140 Sivas, Turkey, e-mail: nhuseyin@cumhuriyet.edu.tr .

Anar Huseyin, Associate Professor, Sivas Cumhuriyet University, Faculty of Science, 58140 Sivas, Turkey, e-mail: ahuseyin@cumhuriyet.edu.tr .

Khalik Garakishi oglu Guseinov, Dr. Phys.-Math. Sci., Professor, Eskisehir Technical University, Faculty of Science, 26470 Eskisehir, Turkey, e-mail: kguseynov@eskisehir.edu.tr .