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CONSTRUCTION OF DISCONTINUOUS PIECEWISE QUADRATIC VALUE FUNCTIONS IN A TARGET CONTROL PROBLEM

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We consider a method for the approximate solution of solvability and control synthesis problems for a nonlinear autonomous system of ordinary differential equations on a fixed time interval. The proposed method is based on hybridization of equations and consideration of equivalent problems for a piecewise linear system. Next, the value function is constructed as an approximate solution of the Hamilton–Jacobi–Bellman equation, and the comparison principle is applied. The solution is chosen from the class of piecewise quadratic functions. To improve the accuracy of the method, the specified value function is assumed to have discontinuities on certain sets in the state space. We propose a numerical algorithm for feedback control calculation and obtain an a priori error estimate of reaching the target set for the original nonlinear system.

Keywords: nonlinear dynamics, control synthesis, dynamic programming, comparison principle, linearization, switched system, piecewise quadratic value function.

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