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CONSTANT-FACTOR APPROXIMATION ALGORITHMS FOR A SERIES OF COMBINATORIAL ROUTING PROBLEMS BASED ON THE REDUCTION TO ASYMMETRIC TRAVELING SALESMAN PROBLEM

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For the first time, algorithms with constant performance guarantees are substantiated for a series of asymmetric routing problems of combinatorial optimization: Steiner Cycle Problem (SCP), Generalized Traveling Salesman Problem (GTSP), Capacitated Vehicle Routing Problem with Unsplittable Client Demands (CVRP-UCD), and Prize Collecting Traveling Salesman Problem (PCTSP). The presented results are united by the property that they all rely on polynomial cost-preserving reduction to appropriate statements of Asymmetric Traveling Salesman Problem (ATSP) and on the $(22+\varepsilon)$-approximate algorithm for this classical problem proposed by O. Svensson and V. Traub in 2019.

Keywords: Asymmetric Traveling Salesman Problem, constant-factor approximation algorithm, polynomial-time reduction, Steiner Cycle Problem, Generalized Traveling Salesman Problem, Prize Collecting Traveling Salesman Problem, Vehicle Routing Problem.

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