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OPTIMAL COMBINATION TREATMENT PROTOCOLS FOR A CONTROLLED MODEL OF BLOOD CANCER

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A combined treatment of blood cancer is considered on a given time interval. The treatment consists of two stages. At the first stage, the patient undergoes therapy that has a powerful effect on the body in order to eliminate the disease. At the second stage, therapy is aimed at maintaining the achieved positive effect. The moment of transition from the first stage of treatment to the second is not fixed and depends on the patient's condition. The implementation of such treatment is mathematically described by a two-dimensional Lotka–Volterra competition model whose variables are the concentrations of healthy and cancerous cells. The model contains two bounded control functions expressing the intensity of applied therapies. The quality of such combined treatment is assessed by minimizing an objective function that describes the dynamics of the concentrations of healthy and cancerous cells at the ends of the first and second stages of the total treatment period. For the theoretical analysis of this optimization problem, the Pontryagin maximum principle for hybrid control systems is applied. The results of numerical calculations performed in the BOCOP-2.2.1 environment are also presented and discussed in detail.

Keywords: blood cancer, two-dimensional Lotka–Volterra competition model, hybrid control system, optimal control, Pontryagin maximum principle.

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