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TRANSFINITE VERSION OF THE PROGRAM ITERATION METHOD IN A CONVERGENCE GAME PROBLEM FOR AN ABSTRACT DYNAMICAL SYSTEM

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The game problem of convergence of motions is considered for an abstract dynamical system with a given target set inside the phase constraints. An arbitrary subset of real numbers acts as a time "interval." The target set $\mathcal{M}$ and the phase constraints $\mathcal{N}$ obey the $\mathcal{M} \subset \mathcal{N}$ embedding. Nonanticipating multifunctions defined on the histories of disturbances are considered as admissible control strategies. A description of the solvability set and the construction of resolving control strategies based on the method of program iterations are given. At the same time, by increasing the "number" of iterations of the program absorption operator, it is possible to expand (compared to the original version of the method) the areas of applicability due to the weakening or complete rejection of the topological requirements to the system dynamics, the target set, and phase constraints. The proposed constructions and their justification use the technique of fixed points of monotone mappings in partially ordered sets.

Keywords: convergence game problem, program iterations, abstract dynamical system, non-anticipating strategies.

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