

MSC: 65K10, 37N40, 93C95

DOI: 10.21538/0134-4889-2022-28-3-17-29

PROJECTION METHOD FOR ECONOMIC GROWTH PROBLEMS ON AN INFINITE TIME INTERVAL

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A projection method is proposed for economic growth problems on an infinite interval. The orthogonal Laguerre polynomials multiplied by an exponential are used as the basis functions for the parametrization of the solution. Convergence of the method has been studied numerically for integrable cases in the Ramsey model. It is shown that the best convergence of the method is achieved if the parameter in the exponent is chosen to be equal to the negative eigenvalue of the linearization matrix of the Hamiltonian system at the equilibrium at infinity. In the considered examples, the projection method leads to a system of equations with a small number of unknowns, in contrast to the methods using finite difference approximation.

Keywords: Galerkin method, Gauss–Laguerre quadrature, infinite-horizon control problem, transversality conditions, Ramsey model, CRRA utility function, Bernoulli transformation.

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Received May 30, 2022

Revised July 24, 2022

Accepted August 1, 2022

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Cite this article as: B. M. Arystanbekov, N. B. Melnikov. Projection method for economic growth problems on an infinite time interval. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2022, vol. 28, no. 3, pp. 17–29.