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POLYNOMIALS LEAST DEVIATING FROM ZERO WITH A CONSTRAINT ON THE LOCATION OF ROOTS

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We consider Chebyshev's problem on polynomials least deviating from zero on a compact set K with a constraint on the location of their roots. More exactly, the problem is considered on the set $\mathcal{P}_n(G)$ of polynomials of degree n that have unit leading coefficient and do not vanish on an open set G. An exact solution is obtained for $K=[-1,1]$ and $G=\{z\in\mathbb{C} : |z|<R\}$, $R\ge \varrho_n$, where ϱ_n is a number such that $\varrho_n^2\le(\sqrt{5}-1)/2$. In the case $\operatorname{Conv} K\subset \overline{G}$, the problem is reduced to similar problems for the set of algebraic polynomials all of whose roots lie on the boundary ∂G of the set G. The notion of Chebyshev constant $\tau(K,G)$ of a compact set K with respect to a compact set G is introduced, and two-sided estimates are found for $\tau(K,G)$.

Keywords: Chebyshev polynomial of a compact set, Chebyshev constant of a compact set; constraints on the roots of a polynomial.

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