TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 28 No. 3 **2022**

MSC: 49N79, 49N70, 91A24

DOI: 10.21538/0134-4889-2022-28-3-129-141

ON A LINEAR GROUP PURSUIT PROBLEM WITH FRACTIONAL DERIVATIVES

A. I. Machtakova, N. N. Petrov

A problem of pursuit of one evader by a group of pursuers is considered in a finite-dimensional Euclidean space. The dynamics is described by the system

$$D^{(\alpha_i)} z_i = A_i z_i + B_i u_i - C_i v, \quad u_i \in U_i, \quad v \in V,$$

where $D^{(\alpha)} f$ is the Caputo derivative of order α of a function f. The sets of admissible controls of the players are convex and compact. The terminal set consists of cylindrical sets M_i of the form $M_i = M_i^1 + M_i^2$, where M_i^1 is a linear subspace of the phase space and M_i^2 is a convex compact set from the orthogonal complement of M_i^1. We propose two approaches to solving the problem, which ensure the termination of the game in a certain guaranteed time in the class of quasi-strategies. In the first approach, the pursuers construct their controls so that the terminal sets "cover" the evader's uncertainty region. In the second approach, the pursuers construct their controls using resolving functions. The theoretical results are illustrated by model examples.

Keywords: differential game, group pursuit, pursuer, evader, fractional derivative.

REFERENCES

1. Isaacs R. *Differential games*. NY: John Wiley & Sons, 1965, 384 p. ISBN: 0471428604 .
2. Krasovskii N.N., Subbotin A.I. *Pozitsionnye differentsial'nye igry* [Positional differential games]. Moscow: Nauka Publ., 1974, 458 p.
3. Pontryagin L.S. *Izbrannye nauchnye trudy, II* [Selected scientific works, II]. Moscow: Nauka Publ., 1988, 575 p. ISBN: 5-02-14410-X .
4. Grigorenko N.L. *Matematicheskie metody upravleniya neskol'kimi dinamicheskimi protsessami* [Mathematical methods of control a few dynamic processes]. Moscow: Moscow State University Publ., 1990, 198 p. ISBN: 5-211-00954-1 .
5. Chikrii A. *Conflict-controlled processes*. Dordrecht: Springer, 1997, 404 p. doi: 10.1007/978-94-017-1135-7 . Original Russian text published in Chikrii A.A. *Konfliktno upravlyaemye processy*, Kiev: Naukova Dumka Publ., 1992, 384 p.
6. Blagodatskikh A.I., Petrov N.N. *Konfliktnoe vzaimodeistvie grupp upravlyaemykh ob'ektov* [Conflict interaction of groups of controlled objects]. Izhevsk: Udmurt State University Publ., 2009, 266 p. ISBN: 978-5-904524-17-3 .
7. Chikrii A.A., Matichin I.I. Game problems for fractional-order linear systems. *Proc. Steklov Inst. Math.*, 2010, vol. 268, no. 1, pp. 54–70. doi: 10.1134/S0081543810050056 .
8. Gomoyunov M. Solution to a zero-sum differential game with fractional dynamics via approximations. *Dyn. Games Appl.*, 2020, vol. 10, no. 2, pp. 417–443. doi: 10.1007/s13235-019-00320-4 .
9. Petrov N.N. Group pursuit problem in a differential game with fractional derivatives, state constraints, and simple matrix. *Diff. Eq.*, 2019, vol. 55, no. 6, pp. 841–848. doi: 10.1134/S0012266119060119 .
10. Petrov N.N., Machtakova A.I. Capture of two coordinated evaders in a problem with fractional derivatives, phase restrictions and a simple matrix. *Izv. IMI UdGU*, 2020, vol. 56, pp. 50–62. doi: 10.35634/2226-3594-2020-56-05 (in Russian).
11. Aubin J.P., Frankowska H. *Set-valued analysis*. Boston, MA: Birkhäuser, 2009, 461 p. doi: 10.1007/978-0-8176-4848-0 .
12. Chikrii A.A., Matichin I.I. An analog of the Cauchy formula for linear systems of arbitrary fractional order. *Dopov. Nats. Akad. Nauk Ukr.*, 2007, no. 1, pp. 50–55 (in Russian).

13. Petrov N.N. Controllability of autonomous systems. *Diff. Uravn.*, 1968, vol. 4, no. 4, pp. 606–617 (in Russian).

Received May 30, 2022
Revised July 7, 2022
Accepted July 11, 2022

Funding Agency: This work was supported by the Russian Science Foundation (project no. 21-71-10070).

Machtakova Alena Igorevna, doctoral student, Udmurt State University, Izhevsk, 426034 Russia; Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia, e-mail: bichurina.alyona@yandex.ru .

Petrov Nikolai Nikandrovich, Dr. Phys.-Math. Sci, Prof., Udmurt State University, Izhevsk, 426034, Russia, Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia, e-mail: kma3@list.ru .

Cite this article as: A. I. Machtakova, N. N. Petrov. On a linear group pursuit problem with fractional derivatives. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2022, vol. 28, no. 3, pp. 129–141.