ON THE BAER–SUZUKI WIDTH OF SOME RADICAL CLASSES

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Let $\sigma = \{\sigma_i \mid i \in I\}$ be a fixed partition of the set of all primes into pairwise disjoint nonempty subsets σ_i. A finite group is called σ-nilpotent if it has a normal σ_i-Hall subgroup for any $i \in I$. Any finite group possesses a σ-nilpotent radical, which is the largest normal σ-nilpotent subgroup. In this note, it is proved that there exists an integer $m = m(\sigma)$ such that the σ-nilpotent radical of any finite group coincides with the set of elements x such that any m conjugates of x generate a σ-nilpotent subgroup. Other possible analogs of the classical Baer-Suzuki theorem are discussed.

Keywords: Baer–Suzuki width, σ-nilpotent group, σ-solvable group, complete class of groups.

REFERENCES

1. Shemetkov L.A. *Formatsii konechnykh grupp* [Formations of finite groups]. Moscow: Nauka Publ., 1978, 271 p.

2. Skiba A.N. On σ-subnormal and σ-permutable subgroups of finite groups. *J. Algebra*, 2015, vol. 436, pp. 1–16. doi: 10.1016/j.jalgebra.2015.04.010 .

3. Skiba A.N. On some results in the theory of finite partially soluble groups. *Comm. Math. Stat.*, 2016, vol. 4, no. 3, pp. 281–309. doi: 10.1007/s40304-016-0088-z .

4. Baer R. Engelsche Elemente Noetherscher Gruppen. *Math. Ann.*, 1957, vol. 133, no. 3, pp. 256–270. doi: 10.1007/BF02547953 .

5. Suzuki M. Finite groups in which the centralizer of any element of order 2 is 2-closed. *Ann. Math.*, 1965, vol. 82, no. 1, pp. 191–212. doi: 10.2307/1970569 .

6. Gordeev N., Grunewald F., Kunyavskii B., Plotkin E. A description of Baer–Suzuki type of the solvable radical of a finite group. *J. Pure Appl. Algebra*, 2009, vol. 213, no. 2, pp. 250–258. doi: 10.1016/j.jpaa.2008.06.006 .

7. Alperin J., Lyons R. On conjugacy classes of p-elements. *J. Algebra*, 1971, vol 19, no. 2, pp. 536–537. doi: 10.1016/0021-8693(71)90086-X .

8. Gorenstein D. *Finite groups.* 2nd ed. NY: Chelsea P. C., 1980, 519 p. ISBN: 0828403015 .

9. Gordeev N., Grunewald F., Kunyavskii B., Plotkin E. Baer–Suzuki theorem for the solvable radical of a finite group. *C. R. Acad. Sci. Paris, Ser. I*, 2009, vol. 347, no. 5-6, pp. 217–222. doi: 10.1016/j.crma.2009.01.004 .

10. Gordeev N., Grunewald F., Kunyavskii B., Plotkin E. From Thompson to Baer–Suzuki: a sharp characterization of the solvable radical. *J. Algebra*, 2010, vol. 323, no. 10, pp. 2888–2904. doi: 10.1016/j.jalgebra.2010.01.032 .

11. Flavell P., Guest S., Guralnick R. Characterizations of the solvable radical. *Proc. Amer. Math. Soc.*, 2010, vol. 138, no. 4, pp. 1161–1170. doi: 10.1090/S0002-9939-09-10066-7 .

12. Guest S. A solvable version of the Baer–Suzuki theorem. *Trans. Amer. Math. Soc.*, 2010, vol. 362, no. 11, pp. 5909–5946. doi: 10.1090/S0002-9947-2010-04932-3 .

13. Yang N., Revin D.O., Vdovin E.P. Baer–Suzuki theorem for the π-radical. *Isr. J. Math.*, 2021, vol. 245, no. 1, pp. 173–207. doi: 10.1007/s11856-021-2209-y .

14. Yang N., Wu Z., Revin D.O. On the sharp Baer–Suzuki theorem for the π-radical: sporadic groups. *Sib. Math. J.*, 2022, vol. 63, no. 2, pp. 387–394. doi: 10.1134/S0037446622020161 .

15. Yang N., Wu Z., Revin D.O., Vdovin E.P. On the sharp Baer–Suzuki theorem for the π-radical. *arXiv*:2105.02442 , 2021, 36 p. Available on: https://arxiv.org/pdf/2105.02442.pdf .

16. Tyutyanov V.N. A criterion of non-simplicity for a finite group. *Izv. Gomel. Gos. Univ. Im. F. Skoriny*, 2000, vol. 16, no. 3, pp. 125–137 (in Russian).

17. Revin D.O. On Baer–Suzuki π-theorems. *Sib. Math. J.*, 2011, vol. 52, no. 2, pp. 340–347. doi: 10.1134/S0037446611020170 .

18. Wielandt H. Zusammengesetzte Gruppen: Hölder Programm heute. In: *The Santa Cruz conf. on finite groups (Santa Cruz, 1979)*, Ser. Proc. Sympos. Pure Math., vol. 37, Providence, RI: Amer. Math. Soc., 1980, pp. 161–173. doi: 10.1090/pspum/037/604575 .

19. *The Kourovka notebook. Unsolved problems in group theory.* Edited by V.D. Mazurov and E.I. Khukhro. 20th ed. Novosibirsk: Russian Academy of Sciences Siberian Division Institute of Mathematics, 2022. Available on: https://kourovkanotebookorg.files.wordpress.com/2022/02/20tkt-3.pdf .

20. Isaacs I.M. *Character theory of finite groups.* Ser. Pure Appl. Math., vol. 359, NY: Acad. Press, 1976, 303 p. ISBN: 978-0-8218-4229-4 .

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