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## ON KOLMOGOROV'S INEQUALITY FOR THE FIRST AND SECOND DERIVATIVES ON THE AXIS AND ON THE PERIOD

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We study the inequality $\|y'\|_{L_q(G)} \leq K(r,p,G)\|y\|^{1/2}_{L_r(G)}\|y''\|^{1/2}_{L_p(G)}$ on the real line $G=\mathbb{R}$ and on the period $\mathbb{T}$ for $q\in[1,\infty)$, $r\in(0,\infty]$, $p\in[1,\infty]$, and $1/r+1/p=2/q$. We prove that the exact constant $K(r,p,\mathbb{R})$ is equal to the exact constant $K_1$ in the inequality $\|u'\|_{L_q[0,1]} \leq K_1\|u\|^{1/2}_{L_r[0,1]}\|u''\|^{1/2}_{L_p[0,1]}$ over the set of convex functions $u(x)$, $x\in[0,1]$, having an absolutely continuous derivative and satisfying the condition $u'(0)=u(1)=0$. As a consequence of this statement, the equality $K(r,p,\mathbb{R})=K(r,p,\mathbb{T})$ established in 2003 by V. F. Babenko, V. A. Kofanov, and S. A. Pichugov for $r\geq 1$, is extended to $r\geq 1/2$. In addition, we give a new proof of the equality $K(r,1,\mathbb{R})=(r+1)^{1/(2(r+1))}$ for $p=1$, $r\in[1,\infty)$, and $q=2r/(r+1)$, which was established by V. V. Arestov and V. I. Berdyshev in 1975.

Keywords: Kolmogorov's inequality, inequalities for norms of functions and their derivatives, exact constants, real axis, period.

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