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ON SHILLA GRAPHS WITH $b = 6$ AND $b_2 \neq c_2$

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A Shilla graph is a distance-regular graph Γ (with valency k) of diameter 3 that has second eigenvalue θ_1 equal to $a = a_3$. In this case a divides k and the parameter $b = b(\Gamma) = k/a$ is defined. A Shilla graph has intersection array $\{ab, (a+1)(b-1), b_2; 1, c_2, a(b-1)\}$. J. Koolen and J. Park showed that for fixed b there are finitely many Shilla graphs. Admissible intersection arrays of Shilla graphs were found for $b \in \{2, 3\}$ by Koolen and Park in 2010 and for $b \in \{4, 5\}$ by A. A. Makhnev and I. N. Belousov in 2021. Makhnev and Belousov also proved the nonexistence of Q-polynomial Shilla graphs with $b = 5$ and found Q-polynomial Shilla graphs with $b = 6$. A Q-polynomial Shilla graph with $b = 6$ has intersection array $\{42t, 5(7t+1), 3(t+3); 1, 3(t+3), 35t\}$ with $t \in \{7, 12, 17, 27, 57\}$, $\{372, 315, 75; 1, 15, 310\}$, $\{744, 625, 125; 1, 25, 620\}$, $\{930, 780, 150; 1, 30, 775\}$, $\{312, 265, 48; 1, 24, 260\}$, $\{624, 525, 80; 1, 40, 520\}$, $\{1794, 1500, 200; 1, 100, 1495\}$, or $\{5694, 4750, 600; 1, 300, 4745\}$. The nonexistence of graphs with intersection arrays $\{372, 315, 75; 1, 15, 310\}$, $\{744, 625, 125; 1, 25, 620\}$, $\{1794, 1500, 200; 1, 100, 1495\}$, and $\{42t, 5(7t+1), 3(t+3); 1, 3(t+3), 35t\}$ was proved earlier. We prove that distance-regular graphs with intersection arrays $\{312, 265, 48; 1, 24, 260\}$, $\{624, 525, 80; 1, 40, 520\}$, and $\{930, 780, 150; 1, 30, 775\}$ do not exist.

Keywords: Shilla graph, distance-regular graph, Q-polynomial graph.

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