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APPROXIMATION OF THE NORMAL TO THE DISCONTINUITY LINES OF A NOISY FUNCTION

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The work is devoted to the construction of regularizing algorithms for solving the ill-posed problem of determining the normal and the position of the discontinuity lines of a function of two variables. It is assumed that the function is smooth outside the discontinuity lines, and at each point on the line it has a discontinuity of the first kind. The case is considered when the exact function is unknown, and instead of it, at each node of a uniform grid with a step τ, the mean values on the square with side τ of the perturbed function are known. The perturbed function approximates the exact function in the space $L_2(\mathbb{R}^2)$ and the perturbation level δ is assumed to be known. Previously, the authors investigated (obtained accuracy estimates for) global discrete regularizing algorithms for approximating the set of discontinuity lines of a noisy function. To suppress noise when constructing the algorithms, the idea of averaging the original disturbed data over both variables is used. In this work, methods are constructed that allow finding a set of pairs (grid point and vector): the grid point approximates the discontinuity line of the exact function, and the corresponding vector approximates the normal to the discontinuity line.These algorithms are investigated for the special case when the break lines are polygonal. Estimates of the accuracy of approximation of discontinuity lines and normals are obtained.

Keywords: ill-posed problem, regularization method, discontinuity lines, global localization, discretization, separability threshold, normal.

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