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HILBERT'S BASIS THEOREM FOR A SEMIRING OF SKEW POLYNOMIALS

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Semirings of skew polynomials are studied. Such semirings are generalizations of both polynomial semirings and skew polynomial rings. Let φ be an endomorphism of a semiring S. The left semiring of skew polynomials over S is the set of polynomials of the form $f = a_0 + a_1x + \ldots + a_kx^k$, $a_i \in S$, with the usual addition and the multiplication given by the rule $xa = \varphi(a)x$. It is known that the semiring of polynomials over a Noetherian semiring does not have to be Noetherian. In 1976, L. Dale introduced the notion of monic ideal of a polynomial semiring $S[x]$ over a commutative semiring, i.e., of an ideal that together with any its polynomial $f = \ldots + ax^k + \ldots$ contains each monomial ax^k. It was shown that the Noetherian property of a semiring S implies the ascending chain condition for the monic ideals from $S[x]$. We study the monic ideals of the semiring of skew polynomials $S[x, \varphi]$. To describe them, we define φ-chains of coefficient sets of ideals from the semiring $S[x, \varphi]$. The main result of the paper is the following fact: if φ is an automorphism, then the semiring S is left (right) Noetherian if and only if $S[x, \varphi]$ satisfies the ascending chain condition for the left (right) monic ideals. Examples are given showing that the injectivity of the endomorphism φ is not sufficient for the validity of the formulated result.

Keywords: semiring of skew polynomials, monic ideal, φ-chain of coefficient sets, Hilbert's basis theorem.

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