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TOMOGRAPHIC CHARACTERIZATIONS OF SUNS IN THREE-DIMENSIONAL SPACES

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Recently A. R. Alimov and B. B. Bednov characterized the three-dimensional spaces in which any Chebyshev set is monotone path-connected. In particular, they showed that any Chebyshev set in a three-dimensional space with cylindrical norm is monotone path-connected. The author of the present paper obtained a similar result for closed sets with continuous (lower semicontinuous) metric projection. R. Aumann established that if the section of a compact subset M of a finite-dimensional space by any hyperplane is acyclic, then M is convex. A sun is considered as a possible generalization of a convex set — it is well known that any point not lying in a sun can be separated from it by an open support cone. In the present paper, we consider the problem of tomographic classification of suns in terms of approximative and geometric properties of their sections by tangent planes. We consider the case of three-dimensional spaces with cylindrical norm. In these spaces, we introduce the notion of a tangent plane, which generalizes the notion of a tangent direction to a sphere introduced by A. R. Alimov and E. V. Shchepin. The results obtained in the paper partially generalize and extend the mentioned studies. We give necessary and sufficient conditions for the monotone path-connectedness of approximatively defined sets in three-dimensional cylindrical spaces in terms of approximative and geometric properties of their sections by tangent planes.

Keywords: best approximation, Chebyshev set, sun, monotone path-connected set.

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