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FINITE SOLVABLE GROUPS WHOSE GRUENBERG–KEGEL GRAPHS ARE ISOMORPHIC TO THE PAW

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The Gruenberg–Kegel graph (or the prime graph) of a finite group G is the graph, in which the vertex set is the set of all prime divisors of the order of G and two different vertices p and q are adjacent if and only if there exists an element of order pq in G. The paw is the graph on four vertices whose degrees are 1, 2, 2, and 3. We consider the problem of describing finite groups whose Gruenberg–Kegel graphs are isomorphic as abstract graphs to the paw. For example, the Gruenberg–Kegel graphs of the groups A_{10} and $\mathrm{Aut}(J_2)$ are isomorphic as abstract graphs to the paw. In this paper, we describe finite solvable groups whose Gruenberg–Kegel graphs are isomorphic as abstract graphs to the paw.

Keywords: finite group, solvable group, Gruenberg–Kegel graph, paw.

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