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ON A CLASS OF VERTEX-PRIMITIVE ARC-TRANSITIVE AMPLY REGULAR GRAPHS

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A simple k-regular graph with v vertices is an amply regular graph with parameters (v, k, λ, μ) if any two adjacent vertices have exactly λ common neighbors and any two vertices which are at distance 2 in this graph have exactly μ common neighbors. Let G be a finite group, $H \leq G$, $\mathfrak{H} = \{H^g \mid g \in G\}$ be the corresponding conjugacy class of subgroups of G, and $1 \leq d$ be an integer. We construct a simple graph $\Gamma(G, H, d)$ as follows. The vertices of $\Gamma(G, H, d)$ are the elements of $\mathfrak{H}$, and two vertices H_1 and H_2 from $\mathfrak{H}$ are adjacent in $\Gamma(G, H, d)$ if and only if $|H_1 \cap H_2| = d$. In this paper we prove that if q is a prime power with $13 \leq q \equiv 1 \pmod 4$, $G = SL_2(q)$, and H is a dihedral maximal subgroup of G of order $2(q-1)$, then the graph $\Gamma(G, H, 8)$ is a vertex-primitive arc-transitive amply regular graph with parameters $\left(\frac{q(q+1)}{2}, \frac{q-1}{2}, 1, 1\right)$ and with $\mathrm{Aut}(PSL_2(q)) \leq \mathrm{Aut}(\Gamma)$. Moreover, we prove that $\Gamma(G, H, 8)$ has a perfect 1-code, in particular, its diameter is more than 2.

Keywords: finite simple group, arc-transitive graph, amply regular graph, edge-regular graph, graph of girth 3, Deza graph, perfect 1-code.

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