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GROUPS SATURATED WITH FINITE SIMPLE GROUPS $L_3(2^n)$ AND $L_4(2^l)$

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Let $\mathfrak{M}$ be a certain set of groups. For a group G, we denote by $\mathfrak{M}(G)$ the set of all subgroups of G that are isomorphic to elements of $\mathfrak{M}$. A group G is said to be saturated with groups from $\mathfrak{M}$ if any finite subgroup of G is contained in some element of $\mathfrak{M}(G)$. We prove that if G is a periodic group or a Shunkov group and G is saturated with groups from the set $\{L_3(2^n), L_4(2^l) \mid n = 1, 2, \ldots; l = 1, \ldots, l_0\}$, where l_0 is fixed, then the set of elements of finite order from G forms a group isomorphic to one of the groups from the set $\{L_3(R), L_4(2^l) \mid l = 1, \ldots, l\}$, where R is an appropriate locally finite field of characteristic 2.

Keywords: periodic group, Shunkov group, saturation of a group with a set of groups.

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