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CAPACITATED FACILITY LOCATION PROBLEM ON TREE-LIKE GRAPHS

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We consider the network Capacitated Facility Location Problem (CFLP) and its special case—the Uniform Capacitated Facility Location Problem (UCFLP), where all facilities have the same capacity. We show that the UCFLP on a star graph is linear-time solvable if every vertex of the star can be either a facility or a client but not both. We further prove that the UCFLP on a star graph is \mathcal{NP} -hard if the facilities and clients can be located at each vertex of the graph. The UCFLP on a path graph is known to be polynomially solvable. We give a version of the known dynamic programming algorithm for this problem with the improved time complexity $\mathcal{O}(m^2n^2)$, where m is the number of facilities and n is the number of clients. For the CFLP on a path graph we propose a pseudo-polynomial time algorithm based on the work of Mirchandani et al. (1996) with improved time complexity $\mathcal{O}(mB)$, where B is the total demand of the clients.

Keywords: Capacitated Facility Location Problem, Uniform Capacitated Facility Location Problem, NP-hard problem, star graph, path graph, polynomial time algorithm, pseudo-polynomial time algorithm, dynamic programming.

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