ON THE SOLVABILITY OF A SYSTEM OF NONLINEAR INTEGRAL EQUATIONS WITH A MONOTONE HAMMERSTEIN TYPE OPERATOR

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A system of nonlinear integral equations with a noncompact monotone Hammerstein type matrix integral operator is studied on the positive half-line. For specific representations of matrix kernels and nonlinearities involved in the system, the class of vector nonlinear integral equations under consideration has applications in various areas of mathematical physics. In particular, such systems arise in the theory of radiative transfer in inhomogeneous media, in the kinetic theory of gases, and in mathematical biology. The existence of a nontrivial componentwise nonnegative and bounded solution is proved. In one important particular case, the integral asymptotic behavior of the constructed solution is also studied. At the end of the paper, specific examples of nonlinearities and matrix kernels that satisfy the conditions of the formulated theorems are given.

Keywords: monotonicity, bounded solution, iterations, convexity, nonlinearity, convergence.

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