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ON Q-POLYNOMIAL SHILLA GRAPHS WITH $b = 4$

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Shilla graphs introduced by J. H. Koolen and J. Park are considered. In the problem of finding feasible intersection arrays of Shilla graphs with a fixed parameter b, Q-polynomial graphs play an important role. For such graphs, the smallest eigenvalue is the minimum possible for the third nonprincipal eigenvalue. Intersection arrays of Q-polynomial graphs were found for $b = 3$ in 2010 by Koolen and Park and for $b \in \{4, 5\}$ in 2018 by Belousov. In particular, it is known that a Q-polynomial Shilla graph with $b = 4$ has intersection array $\{104, 81, 27; 1, 9, 78\}$, $\{156, 120, 36; 1, 12, 117\}$, or $\{20(q-2), 3(5q-9), 2q; 1, 2q, 15(q-2)\}$, where $q = 6, 9, 18$. We prove that distance-regular graphs with intersection arrays $\{80, 63, 12; 1, 12, 60\}$, $\{140, 108, 18; 1, 18, 105\}$, and $\{320, 243, 36; 1, 36, 240\}$ do not exist.

Keywords: Shilla graph, distance-regular graphs, Q-polynomial graph.

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