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ON THE COINCIDENCE OF GRUENBERG–KEGEL GRAPHS OF AN ALMOST SIMPLE GROUP AND A NONSOLVABLE FROBENIUS GROUP

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Let G be a finite group. Its spectrum $\omega(G)$ is the set of all element orders of G. The prime spectrum $\pi(G)$ is the set of all prime divisors of the order of G. The Gruenberg–Kegel graph (or the prime graph) $\Gamma(G)$ is a simple graph whose vertex set is $\pi(G)$, and two distinct vertices p and q are adjacent in $\Gamma(G)$ if and only if $pq \in \omega(G)$. The structural Gruenberg–Kegel theorem implies that the class of finite groups with disconnected Gruenberg–Kegel graphs widely generalizes the class of finite Frobenius groups, whose role in finite group theory is absolutely exceptional. The question of coincidence of Gruenberg–Kegel graphs of a finite Frobenius group and of an almost simple group naturally arises. The answer to the question is known in the cases when the Frobenius group is solvable and when the almost simple group coincides with its socle. In this short note we answer the question in the case when the Frobenius group is nonsolvable and the socle of the almost simple group is isomorphic to $PSL_2(q)$ for some q.

Keywords: finite group, Gruenberg–Kegel graph (prime graph), nonsolvable Frobenius group, almost simple group.

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