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ANALYTIC DIFFUSION WAVES IN A NONLINEAR PARABOLIC "PREDATOR–PREY" MODEL

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We consider a system of two nonlinear degenerate parabolic equations that are nonlinear generalizations of the Fisher–Kolmogorov–Petrovskii–Piskunov equation. This system is the basis for the predator–prey mathematical model. Its interesting peculiarity is that it has solutions of the diffusion (heat, filtration) wave type propagating over a zero background with a finite velocity. This peculiarity is a consequence of nonlinear degeneracy. We consider the problem of constructing a diffusion wave of the system that has a known law of front motion. A theorem of existence and uniqueness of a piecewise analytic solution is proved. The proof is constructive: we find a solution in the form of power series and give recursive formulas for the coefficients. The local convergence is proved by the majorant method. The obtained results follow the tradition of Academician A. F. Sidorov's scientific school to use the power series method to solve degenerate parabolic problems. Note that similar studies were previously conducted for single equations, as well as for reaction–diffusion systems that were significantly simpler in structure than the one mentioned above. The increased complexity makes it impossible to automatically transfer the earlier results to the case under consideration and affects both the construction of the solution and the proof of convergence. The convergence is local, but the obtained exact solutions of traveling wave type can illustrate the behavior of the solution outside the convergence domain. In order to construct the solution, we reduce the original problem to the Cauchy problem for a system of ordinary differential equations. This system is integrated in quadratures, and its solutions are written explicitly. The obtained formulas may be used to verify numerical calculations.

Keywords: nonlinear degenerate parabolic system, predator–prey model, diffusion wave, existence theorem, power series, majorant method, exact solutions.

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