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**STRUCTURAL AND ALGORITHMIC PROPERTIES  
OF MAXIMAL DISSOCIATING SETS IN GRAPHS****O. I. Duginov, B. M. Kuskova, D. S. Malyshev, N. A. Shur**

A subset of the vertex set of a graph is called dissociating if the degrees of the vertices of the subgraph generated by this subset do not exceed 1. A dissociating set is maximal if it is not contained in any dissociating set with a greater number of vertices. Estimates for the greatest (smallest) number of vertices in a maximal dissociating set of a graph are proposed. It is proved that the problem of finding a maximal dissociating set of smallest cardinality is NP-hard for quasichordal bipartite graphs. In addition, it is proved that the problem of finding a maximal dissociating set of smallest cardinality is NP-hard for chordal bipartite graphs, bipartite graphs with the greatest degree of a vertex equal to 3, planar graphs with large girth, and for classes of graphs characterized by finite lists of forbidden generated biconnected subgraphs. A linear algorithm for solving the latter problem in the class of trees is proposed.

Keywords: maximal dissociating set of a graph, problem of finding a maximal generated subgraph with maximum degree of a vertex at most 1, maximal dissociation set, perfect elimination bipartite graph, NP-completeness, hereditary graph classes, trees.

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