REGULARIZATION ANALYSIS OF A DEGENERATE PROBLEM OF IMPULSIVE STABILIZATION FOR A SYSTEM WITH TIME DELAY

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A degenerate problem of stabilizing a linear autonomous system of differential equations with time delay and impulse controls is considered. A nondegenerate criterion for the quality of transient processes, close to a degenerate one, is used for its regularization. The regularized stabilization problem for impulse controls is replaced by an auxiliary non-degenerate optimal stabilization problem for non-impulse controls containing time delay. Bellman's dynamic programming principle is used to solve the auxiliary problem. When finding the governing system of equations for the coefficients of the quadratic Bellman functional, the formulation of the optimal stabilization problem in the functional spaces of states and controls is used. A representation is obtained for the impulse of the optimal stabilizing control. The difficult problem of finding a solution to the governing system of equations for the Bellman functional is replaced by the problem of finding a solution to the governing system of equations for the coefficients of the representation of the optimal stabilizing control. The latter problem has a lower dimension. The asymptotic dependence of the optimal stabilizing control on the regularization parameter is found when the dimension of the control vector coincides with the dimension of the state vector.

Keywords: linear autonomous system, time delay, optimal stabilization, impulse control.

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