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ON THE BEST M -TERM APPROXIMATIONS OF FUNCTIONS FROM THE NIKOL'SKII–BESOV CLASS IN THE LORENTZ SPACE

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We consider spaces of periodic functions of many variables, specifically, the Lorentz space $L_{p,\tau}(\mathbb{T}^m)$ and the Nikol'skii–Besov space $S_{p,\tau,\theta}^r B$, and study the best M -term approximation of a function $f \in L_{p,\tau}(\mathbb{T}^m)$ by trigonometric polynomials. Order-exact estimates for the best M -term approximations of functions from the Nikol'skii–Besov class $S_{p,\tau_1,\theta}^r B$ in the norm of the space $L_{q,\tau_2}(\mathbb{T}^m)$ are derived for different relations between the parameters p, q, τ_1, τ_2 , and θ .

Keywords: Lorentz space, Nikol'skii–Besov class, trigonometric polynomial, best M -term approximation.

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