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ASYMPTOTICS OF A SOLUTION TO A TIME-OPTIMAL CONTROL PROBLEM WITH AN UNBOUNDED TARGET SET IN THE CRITICAL CASE

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We study a time-optimal control problem for a singularly perturbed linear autonomous system with smooth geometric constraints on the control in the form of a ball and an unbounded target set:

$$\begin{cases} \dot{x} = y, & x,\, y \in \mathbb{R}^{2m}, \quad u \in \mathbb{R}^{2m}, \\ \varepsilon^2 \dot{y} = Jy + u, & \|u\| \leqslant 1, \quad 0 < \varepsilon \ll 1, \\ x(0) = x^0 \neq 0, \quad y(0) = y^0, \\ x(T_\varepsilon) = 0, \quad T_\varepsilon \longrightarrow \min, \end{cases}$$

where $J = \begin{pmatrix} 0 & I_m \\ 0 & 0 \end{pmatrix}$. The main difference of this case from the systems with fast and slow variables studied earlier is that here the matrix at the fast variables is a multidimensional analog of the second-order Jordan cell with zero eigenvalue, and thus does not satisfy the standard condition of asymptotic stability. The solvability of the problem is proved. The main system of equations for finding a solution is written. In the case $m = 1$, we derive and justify a complete asymptotics in the sense of Poincaré with respect to the asymptotic sequence $\varepsilon^q \ln^p \varepsilon$, $q \in \mathbb{N}$, $q - 1 \geqslant p \in \mathbb{N} \cup \{0\}$, of the optimal time and of the vector generating the optimal control.

Keywords: optimal control, time-optimal control problem, unbounded target set, singularly perturbed problem, asymptotic expansion, small parameter.

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