TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 28 No. 1 **2022**

MSC: 34B45, 34L05, 34E15, 34E05

DOI: 10.21538/0134-4889-2022-28-1-40-57

TAYLOR SERIES FOR RESOLVENTS OF OPERATORS ON GRAPHS WITH SMALL EDGES

D. I. Borisov, L. I. Gazizova

We consider a second-order elliptic self-adjoint operator on a graph with small edges. Such a graph is obtained by compressing a given graph by a factor of ε^{-1} and then gluing it to another fixed graph; here ε is a small positive parameter. No significant constraints are imposed on this pair of graphs. On such a graph, a general second-order self-adjoint elliptic operator is specified; its differential expression contains derivatives of all orders with variable coefficients and a variable potential. The boundary conditions at the vertices of the graph are also chosen in a general form. All coefficients both in the differential expression and in the boundary conditions can additionally depend on the small parameter ε; this dependence is assumed to be analytical. As was established earlier, the parts of the resolvent of the operator corresponding to the restrictions of the resolvent to the edges of fixed length and to the small edges are analytic in ε as operators in the corresponding spaces, and the restriction to the small edges should be additionally wrapped by a pair of expansion operators. Analyticity means the possibility to represent these operators in the form of the corresponding Taylor series. The first main result of the paper is a procedure similar to the matching of asymptotic expansions for the recursive determination of all coefficients of these Taylor series. The second main result is the representation of the resolvent by a convergent series similar to a Taylor series with effective estimates of the residuals.

Keywords: graph, small edge, elliptic operator, resolvent, analyticity, Taylor series, matching of asymptotic expansions.

REFERENCES

1. Pokornyi Yu.V., Penkin O.M., Pryadiev V.L., Borovskikh A.V., Lazarev K.P., Shabrov S.A. *Differentsial'nye uravneniya na geometricheskikh grafakh* [Differential equations on geometric graphs]. Moscow: Fizmatlit Publ., 2005, 272 p. ISBN: 5-9221-0425-X .
2. Berkolaiko G., Kuchment P. *Introduction to quantum graphs.* Providence: Americ. Math. Soc., 2013, 270 p. ISBN: 0821892118 .
3. Cheon C., Exner E., Turek T. Approximation of a general singular vertex coupling in quantum graphs. *Ann. Phys.*, 2010, vol. 325, no. 3, pp. 548–578. doi: 10.1016/j.aop.2009.11.010 .
4. Zhikov V.V. Homogenization of elasticity problems on singular structures. *Izv. Math.*, 2002, vol. 66, no. 2, pp. 299–365. doi: 10.1070/IM2002v066n02ABEH000380 .
5. Berkolaiko B., Latushkin L., Sukhtaiev S. Limits of quantum graph operators with shrinking edges. *Advances in Mathematics*, 2019, vol. 352, pp. 632–669. doi: 10.1016/j.aim.2019.06.017 .
6. Cacciapuoti C. Scale invariant effective hamiltonians for a graph with a small compact core. *Symmetry*, 2019, vol. 11, no. 3, art. no. 359. doi: 10.3390/sym11030359 .
7. Borisov D.I., Konyrkulzhaeva M.N. Perturbation of threshold of the essential spectrum of the Schrödinger operator on the simplest graph with a small edge. *J. Math. Sci.*, 2019, vol. 239, no. 3, pp. 248–267. doi: 10.1007/s10958-019-04302-0 .
8. Borisov D.I., Mukhametrakhimova A.I. On a model graph with a loop and small edges. *J. Math. Sci.*, 2020, vol. 251, no. 5, pp. 573–601. doi: 10.1007/s10958-020-05118-z .
9. Borisov D.I., Konyrkulzhaeva M.N., Mukhametrakhimova A.I. On discrete spectrum of a model graph with loop and small edges. *J. Math. Sci.*, 2021, vol. 257, no. 5, pp. 551–568. doi: 10.1007/s10958-021-05503-2 .
10. Maz'ya V., Nazarov S., Plamenevskij B. *Asymptotic theory of elliptic boundary value problems in singularly perturbed domains.* Basel: Birkhäuser, 2000, 758 p. ISBN: 978-3-7643-2964-8 .

11. Il'in A.M. *Matching of asymptotic expansions of solutions of boundary value problems.* Providence: American Mathematical Society, 1992, Ser. Trans. Math. Monogr., vol. 102, 281 p. ISBN: 978-0-8218-4561-5 . Original Russian text published in Il'in A.M. *Soglasovanie asimtoticheskikh razlozhenii reshenii kraevykh zadach.* Moscow: Nauka Publ., 1989, 336 p.
12. Borisov B. Analyticity of resolvents of elliptic operators on quantum graphs with small edges. *Adv. Math.*, 2021, art. no. 108125. doi: 10.1016/j.aim.2021.108125 .
13. Borisov D.I. Spectra of elliptic operators on quantum graphs with small edges. *Mathematics*, 2021, vol. 9, no. 16, art. no. 1874. doi: 10.3390/math9161874 .
14. Berkolaiko G., Kuchment P. Dependence of the spectrum of a quantum graph on vertex conditions and edge lengths. In: *Spectral Geometry*, Proc. Sympos. Pure Math., vol. 84. Providence, RI: Amer. Math. Soc., 2012, ISBN: 978-0-8218-5319-1 , pp. 117–137.

Received November 30, 2021
Revised December 17, 2021
Accepted December 27, 2021

Funding Agency: This work was supported by the Russian Science Foundation (project no. 20-11-19995).

Denis Ivanovich Borisov, Dr. Phys.-Math. Sci., Prof., Institute of Mathematics of Ufa Federal Research Center of Russian Academy of Sciences, Ufa, 450008 Russia, e-mail: borisovdi@yandex.ru .

Lejsan Ildarovna Gazizova, doctoral student, Institute of Mathematics of Ufa Federal Research Center of Russian Academy of Sciences, Ufa, 450008 Russia, e-mail: gazizovalejsa@gmail.com .

Cite this article as: D. I. Borisov, L. I. Gazizova. Taylor series for resolvents of operators on graphs with small edges, *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2022, vol. 28, no. 1, pp. 40–57 .