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ON THE WEISS CONJECTURE. I

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Let Γ be a connected finite graph and G a vertex-transitive group of automorphisms of Γ such that the stabilizer G_x in G of a vertex x of Γ induces on the neighborhood $\Gamma(x)$ of x a primitive permutation group $G_x^{\Gamma(x)}$. The Weiss conjecture says that, under this assumption, the order of G_x is bounded from above by a number depending only on the degree $|\Gamma(x)|$ of Γ. In the work whose first part is the present paper we show that some results of the theory of finite groups can be used to provide unified considerations of a number of cases of the Weiss conjecture (including a number of cases not considered before). Although this first part is introductory, it makes possible to use certain previous results to confirm the Weiss conjecture for all primitive groups $G_x^{\Gamma(x)}$ different from groups of AS type and from groups of PA type (constructed on the basis of groups of AS type).

Keywords: graph, group of automorphisms, Weiss conjecture.

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