TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 28 No. 1 **2022**

MSC: 20E28, 20F50

DOI: 10.21538/0134-4889-2022-28-1-239-246

ON PERIODIC COMPLETELY SPLITTABLE GROUPS

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We study an infinite periodic group G with involutions that coincides with the set-theoretic union of a collection of proper locally cyclic subgroups with trivial pairwise intersections. It is proved that if G contains an elementary subgroup E_8, then either G is locally finite (and its structure is described) or its subgroup $O_2(G)$ is elementary and strongly isolated in G. If G has a finite element of order greater than 2 and the 2-rank of G is not 2, then G is locally finite, and its structure is described.

Keywords: periodic group, completely splittable group, 2-rank of a group, strongly isolated subgroup, finite element.

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Received October 10, 2021
Revised December 16, 2021
Accepted December 20, 2021

Funding Agency: This work was supported by the Russian Foundation for Basic Research (project no. 19-01-00566 A) and by the Krasnoyarsk Mathematical Center, which is financed by the Ministry of Science and Higher Education of the Russian Federation within the project for the creation and development of regional centers for mathematical research and education (agreement no. 075-02-2020-1534/1).

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Cite this article as: A. I. Sozutov. On periodic completely splittable groups, *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2022, vol. 28, no. 1, pp. 239–246 .