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ON THE \mathfrak{F} -NORM OF A FINITE GROUP

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Let G be a finite group, and let \mathfrak{F} be a nonempty formation. Then the intersection of the normalizers of the \mathfrak{F} -residuals of all subgroups of G is called the \mathfrak{F} -norm of G and is denoted by $N_{\mathfrak{F}}(G)$. A group G is called \mathfrak{F} -critical if $G \notin \mathfrak{F}$, but $U \in \mathfrak{F}$ for any proper subgroup U of G . We say that a finite group G is generalized \mathfrak{F} -critical if G contains a normal subgroup N such that $N \leq \Phi(G)$ and the quotient group G/N is \mathfrak{F} -critical. In this publication we prove the following result: *If G does not belong to the nonempty hereditary formation \mathfrak{F} , then the \mathfrak{F} -norm $N_{\mathfrak{F}}(G)$ of G coincides with the intersection of the normalizers of the \mathfrak{F} -residuals of all generalized \mathfrak{F} -critical subgroups of G . In particular, the norm $N(G)$ of G coincides with the intersection of the normalizers of all cyclic subgroups of G of prime power order.*

Keywords: finite group, hereditary formation, \mathfrak{F} -residual of a group, \mathfrak{F} -norm of a group, generalized \mathfrak{F} -critical group.

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