

MSC: 20B07, 20F50

DOI: 10.21538/0134-4889-2022-28-1-218-231

AT-GROUPS THAT ARE NOT AT-SUBGROUPS: TRANSITION FROM AT_ω -GROUPS TO AT_Ω -GROUPS

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Periodic nonlocally finite (Burnside) groups of infinite period are studied. The first explicit example of such groups was proposed by S.V. Aleshin in 1972. His construction was generalized to AT-groups, i.e., tree automorphism groups. A number of known problems have been solved with the help of AT-groups. Up to now, in reality, only the class of AT_ω -groups, i.e., the class of AT-groups over a sequence of cyclic groups of prime order, has been studied. In this paper, the class of AT_Ω -groups, i.e., of AT-groups over a sequence of cyclic groups of arbitrary finite order, is studied. The difference between AT_ω -groups and true AT_Ω -groups was revealed by the solution of the Kourovka Problem 16.79. The study of the class of AT_Ω -groups has allowed us to introduce a number of new notions. Now the AT_ω -groups can be considered as elementary AT-groups by which the AT-groups over a sequence of periodic groups are saturated. We propose a strategy for studying such AT-groups and give promising directions of this kind of research.

Keywords: Burnside groups, residually finite group, finiteness conditions, AT-groups, trees, wreath product.

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Received November 11, 2021

Revised January 18, 2021

Accepted January 24, 2021

Funding Agency: This work was supported by the Scholarship Program of the Vladimir Potanin Foundation.

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Cite this article as: A. V. Rozhkov. AT-groups that are not AT-subgroups: Transition from AT_ω -groups to AT_Ω -groups, *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2022, vol. 28, no. 1, pp. 218–231.