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**ON THE CALCULATION OF THE HAUSDORFF DEVIATION
OF CONVEX POLYGONS
IN \mathbb{R}^2 FROM THEIR GEOMETRIC DIFFERENCE WITH DISKS**

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We study a problem concerning the calculation of the Hausdorff deviation of convex polygons in \mathbb{R}^2 from their geometric difference with disks of sufficiently small radius. Problems of this kind, in which not only convex polygons but also convex compact sets in Euclidean space \mathbb{R}^n are considered, arise in various fields of mathematics, in particular, in the theory of differential games, control theory, and convex analysis. Estimates of the Hausdorff deviations of convex compact sets in \mathbb{R}^n from their geometric difference with closed balls in \mathbb{R}^n are found in the works of L.S. Pontryagin and his colleagues. These estimates are essential in deriving an estimate for the discrepancy between Pontryagin's alternating integral in linear differential games of pursuit and alternating sums. Similar estimates turn out to be useful in deriving an estimate for the discrepancy between reachable sets of nonlinear control systems in \mathbb{R}^n and the sets approximating them. The paper considers a convex polygon in \mathbb{R}^2 . We derive a formula for the Hausdorff deviation of the polygon from its geometric difference with a disk in \mathbb{R}^2 whose radius is less than the smallest of the radii of the circles inscribed in the three-links of the polygon.

Keywords: convex polygon in \mathbb{R}^2 , Hausdorff deviation, disk, geometric difference of sets.

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