TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 28 No. 1 **2022**

MSC: 05E30, 05C50

DOI 10.21538/0134-4889-2022-28-1-199-208

INVERSE PROBLEMS IN THE CLASS OF DISTANCE-REGULAR GRAPHS OF DIAMETER 4

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For a distance-regular graph Γ of diameter 4, the graph $\Delta=\Gamma_{1,2}$ can be strongly regular. In this case, the graph $\Gamma_{3,4}$ is strongly regular and complementary to Δ. Finding the intersection array of Γ from the parameters of $\Gamma_{3,4}$ is an inverse problem. In the present paper, the inverse problem is solved in the case of an antipodal graph Γ of diameter 4. In this case, $r=2$ and $\Gamma_{3,4}$ is a strongly regular graph without triangles. Further, Γ is an $AT4(p,q,r)$-graph only in the case $q=p+2$ and $r=2$. Earlier the authors proved that an $AT4(p,p+2,2)$-graph does not exist. A Krein graph is a strongly regular graph without triangles for which the equality in the Krein bound is attained (equivalently, $q_{22}^2=0$). A Krein graph $\mathrm{Kre}(r)$ with the second eigenvalue r has parameters $((r^2+3r)^2, r^3+3r^2+r, 0, r^2+r)$. For the graph $\mathrm{Kre}(r)$, the antineighborhood of a vertex is strongly regular with parameters $((r^2+2r-1)(r^2+3r+1), r^3+2r^2, 0, r^2)$ and the intersection of the antineighborhoods of two adjacent vertices is strongly regularly with parameters $((r^2+2r)(r^2+2r-1), r^3+r^2-r, 0, r^2-r)$. Let Γ be an antipodal graph of diameter 4, and let $\Delta=\Gamma_{3,4}$ be a strongly regular graph without triangles. In this paper it is proved that Δ cannot be a graph with parameters $((r^2+2r-1)(r^2+3r+1), r^3+2r^2, 0, r^2)$, and, if Δ is a graph with parameters $((r^2+2r)(r^2+2r-1), r^3+r^2-r, 0, r^2-r)$, then $r>3$. It is proved that a distance-regular graph with intersection array $\{32,27,12(r-1)/r,1;1,12/r,27,32\}$ exists only for $r=3$, and, for a graph with array $\{96,75,32(r-1)/r,1;1,32/r,75,96\}$, we have $r=2$.

Keywords: distance-regular graph, antipodal graph, graph Γ with strongly regular graph $\Gamma_{i,j}$.

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Received September 14, 2021
Revised January 19, 2022
Accepted January 24, 2022

Funding Agency: This work was supported by the Russian Foundation for Basic Research — the National Natural Science Foundation of China (project no. 20-51-53013).

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Cite this article as: A. A. Makhnev, D. V. Paduchikh. Inverse problems in the class of distance-regular graphs of diameter 4, *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2022, vol. 28, no. 1, pp. 199–208 .