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ON THE EXPONENTS OF COMMUTATORS FROM P. HALL'S COLLECTION FORMULA

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Let G be a group, and let $x, y \in G$. We find an explicit form of the exponents of some commutators from P. Hall's collection formula for the expression $(xy)^n$, $n \in \mathbb{N}$. The exponents for the series of commutators $[y, ux, vy]$ and $[[y, ux], [y, vx]]$ are found in the Hall form, i.e., in the form of integer-valued polynomials in n with zero constant term, and also modulo n when n is a prime number. The exponents for the series of commutators $[[y, ux, vy], t_1[y, u_1x, v_1y], \dots, t_h[y, u_hx, v_hy]]$ are found in the form of multiple combinatorial sums. As a consequence, we obtain an explicit form of Hall's collection formula in two cases: the group G has solvability length 2, the commutator subgroup G' has nilpotency class 2, and $y \in C_G(G')$. A collection formula for the expression $(xy)^n$ is obtained in an explicit form when the group G has solvability length 3. To obtain these results we parameterize the uncollected part of the collection formula by the binary weight function. The results may be useful in solving problems in combinatorial group theory and in studying the regularity of finite p -groups.

Keywords: collection process, collection formula, commutator.

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