Vol. 28 No. 1 **2022**

MSC: 17D99, 16K20, 15A04, 51E15

DOI: 10.21538/0134-4889-2022-28-1-164-181

THE SPREAD SET METHOD FOR THE CONSTRUCTION OF FINITE QUASIFIELDS

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The weakening of the field axioms leads to more general algebraic systems such as near-fields, semifields, and quasifields. The tools for studying these systems are more difficult to use. The spread set method is based on recording multiplication in a quasi-field as a linear transform in the associated linear space. The transition to matrix operations enables the effective application of the method for studying the finite translation planes and their coordinatizing quasifields. We obtain a characteristic property of a spread set for a near-field of dimension two over the kernel. The result is applied to two non-isomorphic near-fields of order 25 and quasifields of order 9. The existence of quasifields with a multiplicative Moufang loop is also discussed. It is proved by the spread set method that a non-associative Moufang quasifield of order 25 does not exist. We list some questions of the theory of finite semifields and semifield projective planes where the spread set method may be useful. This method is also effective in computer constructions of quasifields and translation planes.

Keywords: quasifield, near-field, semifield, spread set, translation plane.

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Received November 10, 2021
Revised December 20, 2021
Accepted December 27, 2021

Funding Agency: This work was supported by the Russian Foundation for Basic Research (project no. 19-01-00566 A).

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Cite this article as: O. V. Kravtsova, D. S. Skok. The spread set method for the construction of finite quasifields, *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2022, vol. 28, no. 1, pp. 164–181 .