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ON THE PRODUCT OF OPERATOR EXPONENTIALS

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A linear densely defined operator A and a domain lying in its regular set and containing the nonpositive real semiaxis are given in a Banach space. A power bound for the norm of the resolvent of the operator at infinity is assumed to be known. The operators e^{tA} ($t \in \mathbb{R}$), given by the corresponding series, and $(e^{tA})_I$ for $t < 0$, introduced on the basis of the integral Cauchy formula, are considered. The question of invertibility of the operator exponentials and the multiplicative property of these exponentials are studied. The operator exponentials can be used for the construction of operator functions of a wider class than that considered by the authors earlier.

Keywords: operator exponent, operator functions, multiplicative property.

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