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ASYMPTOTICS OF A DYNAMIC SADDLE-NODE BIFURCATION FOR THE NUCLEAR SPIN MODEL IN AN ANTIFERROMAGNET

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A system of two nonlinear differential equations with slowly varying coefficients is considered. The system corresponds to one of the models of nuclear spins in antiferromagnets. When written in slow time, the equations contain a small parameter at the derivatives. In the leading terms of the asymptotics with respect to the small parameter, the problem is reduced to a system of algebraic equations. Their roots depend on the slow time. We study solutions whose asymptotics is restructured from one root to another. Such restructuring occurs under a suitable change in the coefficients of the original equations and is identified with a dynamic saddle-node bifurcation. A narrow transition layer appears near the moment of transition (bifurcation), where the solution changes rapidly. The main results are related to the construction of the asymptotics with respect to the small parameter in this layer. To construct the asymptotics, the matching method using three scales is used.

Keywords: equilibrium, dynamic bifurcation, small parameter, asymptotics.

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