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# SEMIGROUPS OF OPERATORS RELATED TO STOCHASTIC PROCESSES IN AN EXTENSION OF THE GELFAND–SHILOV CLASSIFICATION

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Semigroups of operators corresponding to stochastic Levy processes are considered, and their connection with pseudo-differential ($\Psi D$) operators is studied. It is shown that the semigroup generators are $\Psi D$-operators and operators with kernels from the space of slowly growing distributions. A classification of Cauchy problems is constructed for equations with operators from a special class of $\Psi D$-operators with polynomially bounded symbols. The constructed classification extends the Gelfand–Shilov classification for differential systems. In the extended classification, Cauchy problems with generators corresponding to Levy processes are well-posed in the sense of Petrovskii.

Keywords: Levy process, transition probability, semigroup of operators, pseudo-differential operator, Levy–Khintchine formula.

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