TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 27 No. 4 **2021**

MSC: 65J22, 68U10

DOI: 10.21538/0134-4889-2021-27-4-5-18

ALGORITHMS FOR LOCALIZING DISCONTINUITY LINES WITH A NEW TYPE OF AVERAGING

A. L. Ageev, T. V. Antonova

We consider the ill-posed problem of localizing (finding the position of) the discontinuity lines of a function of two variables. It is assumed that the function is smooth outside the discontinuity lines and has a discontinuity of the first kind at each point of these lines. The average values of the perturbed function on a square $\tau \times \tau$ are assumed to be known at each node of a uniform grid with step τ. The perturbed function with a given perturbation level δ approximates the exact function in the space $L_2(\mathbb{R}^2)$. Global discrete regularizing algorithms are constructed for the localization of the discontinuity lines from noisy data. A new approach to the construction of averaging methods for solving the localization problem is proposed. The use of a new type of averaging allows one to construct regularizing algorithms without using the derivative of the averaging function. A new technique is developed and used for deriving estimates. This technique is applicable to a wide range of new methods with a nonclassical averaging domain. On classes of functions with piecewise linear discontinuity lines, estimates of the localization error and other important characteristics of the regularizing algorithm are obtained. It is shown that the new algorithms in some situations are more economical in terms of the number of operations compared to the methods that were investigated by the authors in previous works.

Keywords: ill-posed problem, regularization method, discontinuity lines, global localization, discretization, separability threshold.

REFERENCES

1. Tikhonov A.N., Arsenin V.Ya. *Metody resheniya nekorrektnykh zadach* [Methods for the solution of ill-posed problems]. Moscow: Nauka Publ., 1979, 288 p.
2. Ivanov V.K., Vasin V.V., Tanana V.P. *Theory of linear ill-posed problems and its applications*. Inverse and Ill-Posed Problems Series, Utrecht: VSP, 2002, 281 p. ISBN: 90-6764-367-X/hbk . Original Russian text published in Ivanov V.K., Vasin V.V., Tanana V.P. *Teoriya lineinykh nekorrektnykh zadach i ee prilozheniya*, Moscow: Nauka Publ., 1978, 206 p.
3. Vasin V.V., Ageev A.L. Ill-posed problems with a priori information. Utrecht: VSP, 1995, 255 p. ISBN: 9789067641913 .
4. Mallat S. *A wavelet tour of signal processing: the sparse way*. NY: Acad. Press, 1999, 620 p. ISBN: 0-12-466606-X . Translated to Russian under the title *Veivlety v obrabotke signalov*, Moscow: Mir Publ., 2005, 671 p.
5. Furman Ya.A. (ed.). *Vvedenie v konturnyi analiz i ego prilozheniya k obrabotke izobrazhenii i signalov* [Introduction to contour analysis and its application to image and signal processing]. Moscow: Fizmatlit Publ., 2002, 596 p. ISBN: 5-9221-0255-9 .
6. Gonzalez R.C., Woods R.E. *Digital image processing*, 3rd Ed., NJ: Pearson Prentice Hall, 2006, 976 p. ISBN: 978-0131687288 . Translated to Russian under the title *Tsifrovaya obrabotka izobrazhenii*, Izdanie 3-e ispravlennoe i dopolnennoe, Moscow: Tekhnosfera Publ., 2012, 1104 p.
7. Antonova T.V. Localization method for lines of discontinuity of an approximately defined function of two variables. *Numer. Analys. Appl.*, 2012, vol. 5, no. 4, pp. 285–296. doi: 10.1134/S1995423912040015 .
8. Ageev A.L., Antonova T.V. On the problem of global localization of discontinuity lines for a function of two variables. *Proc. Steklov Inst. Math.*, 2019, vol. 307, no. 1, pp. 1–12. doi: 10.1134/S0081543819070010 .
9. Mafi M., Rajaei H., Cabrerizo M., and Adjouadi M. A robust edge detection approach in the presence of high impulse noise intensity through switching adaptive median and fixed weighted mean filtering. *IEEE Transactions on image processing*, 2018, vol. 27, no. 11, pp. 5475–5489. doi: 10.1109/TIP.2018.2857448 .
10. Al-nasrawi M., Deng G., Thai B. Edge-aware smoothing through adaptive interpolation. *Signal, Image and Video Processing*, 2018, vol. 12, pp. 347–354. doi: 10.1007/s11760-017-1164-x .

11. Chochia P.A. Contour-limited smoothing preserving image structure. *Informatsionnyye protsessy*, 2020, vol. 20, no. 3. pp. 193–204 (in Russian).
12. Ageev A.L., Antonova T.V. New methods for the localization of discontinuities of the first kind for functions of bounded variation. *J. Inverse Ill-Posed Probl.*, 2013, vol. 21, no. 2, pp. 177–191. doi: 10.1515/jip-2012-0039 .
13. Fikhtenholtz G.M. *Kurs differentsial'nogo i integral'nogo ischisleniya* [Differential and Integral Calculus Course]. Vol. 1. Ed. 8, Moscow: Fizmatlit Publ., 2003, 680 p. ISBN: 5-9221-0156-0 .
14. Fikhtengol'ts G.M. *Kurs differentsial'nogo i integral'nogo ischisleniya* [Differential and Integral Calculus Course]. Vol. 3. Ed. 8, Moscow: Fizmatlit Publ., 2003, 728 p. ISBN: 5-9221-0158-7 .

Received March 19, 2021
Revised May 13, 2021
Accepted May 17, 2021

Alexander Leonidivich Ageev, Dr. Phys.-Math. Sci., Prof., Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia, e-mail: ageev@imm.uran.ru .

Tatiana Vladimirovna Antonova, Dr. Phys.-Math. Sci., Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia, e-mail: tvantonova@imm.uran.ru .

Cite this article as: A. L. Ageev, T. V. Antonova. Algorithms for localizing discontinuity lines with a new type of averaging, *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2021, vol. 27, no. 4, pp. 5–18 .