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ORDER ESTIMATES FOR LEBESGUE CONSTANTS OF FOURIER SUMS IN ORLICZ SPACES

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We consider the problem of order estimates for partial sums of trigonometric Fourier series as operators from Orlicz spaces $L^{\varphi}_{2\pi}$ to the space of 2π-periodic continuous functions $C_{2\pi}$. It is established that an arbitrary function φ generating an Orlicz class satisfies the estimate

$$||S_n(f)||_{C_{2\pi}} \le C\varphi^{-1}(n)\ln(n+1)||f||_{L^{\varphi}_{2\pi}}, \qquad (*)$$

where $f \in L^{\varphi}_{2\pi}$, $n \in \mathbb{N}$, $S_n(f)$ is the nth partial sum of the trigonometric Fourier series of f, and the constant $C > 0$ is independent of f and n. In addition, it is shown that if the function φ satisfies the Δ_2-condition, then the estimate can be improved. More exactly,

$$||S_n(f)||_{C_{2\pi}} \le C\varphi^{-1}(n)||f||_{L^{\varphi}_{2\pi}}, \qquad f \in L^{\varphi}_{2\pi},\ n \in \mathbb{N},\ C = C(\varphi). \qquad (**)$$

Counterexamples are constructed, which show that if φ satisfies the Δ_2-condition, then estimate $(**)$ is unimprovable in order on the space $L^{\varphi}_{2\pi}$ and, if φ satisfies the Δ^2-condition, then estimate $(*)$ is unimprovable in order on the space $L^{\varphi}_{2\pi}$.

Keywords: Fourier series, Orlicz space, Lebesgue constants.

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