

MSC: 20D05, 20D60

DOI: 10.21538/0134-4889-2021-27-4-269-275

ON SOME CONJECTURES RELATED TO QUANTITATIVE
CHARACTERIZATIONS OF FINITE NONABELIAN SIMPLE GROUPS¹

Jinbao Li and Wujie Shi

In this note we provide some counterexamples for the conjecture of Moretó on finite simple groups, which says that any finite simple group G can be determined in terms of its order $|G|$ and the number of elements of order p , where p the largest prime divisor of $|G|$. A new characterization of all sporadic simple groups and alternating groups is given. Some related conjectures are also discussed.

Keywords: Finite simple groups, quantitative characterization, the largest prime divisor.

REFERENCES

1. Anabanti C.S. A counterexample to Zarrin's conjecture on sizes of finite nonabelian simple groups in relation to involution sizes. *Arch. Math.*, 2019, vol. 112, no. 3, pp. 225–226. doi: 10.1007/s00013-018-1265-y.
2. Anabanti C.S., Hammer S., Okoli N.C. An infinitude of counterexamples to Herzog's conjecture on involutions in simple groups. *Communications in Algebra*, 2021, vol. 49, no. 4, pp. 1415–1421. doi: 10.1080/00927872.2020.1836563.
3. Bi J.X. Characterization of alternating groups by orders of normalizers of Sylow subgroups. *Algebra Colloq.*, 2001, vol. 8, no. 3, pp. 249–256.
4. Cao H.P., Shi W.J. Pure quantitative characterization of finite projective special unitary groups. *Sci. China Ser. A-Math.*, 2002, vol. 45, no. 6, pp. 761–772. doi: 10.1360/02ys9083.
5. Chen G.Y. On Thompson's conjecture. *J. Algebra*, 1996, vol. 185, no. 1, pp. 184–193. doi: 10.1006/jabr.1996.0320.
6. Conway J.H., Curtis R.T., Norton S.P., Parker R.A., Wilson R.A. *Atlas of finite groups*. Oxford: Clarendon Press, 1985, 252 p. ISBN: 0198531990.
7. Herzog Marcel. On the classification of finite simple groups by the number of involutions. *Proc. Am. Math. Soc.*, 1979, vol. 77, no. 3, pp. 313–314. doi: 10.2307/2042177.
8. Khosravi A., Khosravi B. Two new characterizations for sporadic simple groups. *Pure Math. Appl.*, 2005, vol. 16, no. 3, pp. 287–293.
9. Malinowska I.A. Finite groups with few normalizers or involutions. *Arch. Math.*, 2019, vol. 112, no. 5, pp. 459–465. doi: 10.1007/s00013-018-1290-x.
10. Moretó A. The number of elements of prime order. *Monatsh. Math.*, 2018, vol. 186, no. 1, pp. 189–195. doi: 10.1007/s00605-017-1021-6.
11. Robinson D.J.S. *A course in the theory of groups*. NY; Heidelberg; Berlin: Springer-Verlag, 1982, 481 p. ISBN: 0387906002.
12. Shi W.J. A new characterization of the sporadic simple groups. In: *Group Theory — Proc. Singapore Group Theory Conf. 1987*, Berlin; NY: Walter de Gruyter, 1989, ISBN: 0899254063, pp. 531–540.
13. Shi W.J., Bi J.X. A characteristic property for each finite projective special linear group. In: Kovács L.G. (ed.), *Groups—Canberra 1989*. Lecture Notes in Mathematics, vol. 1456, Berlin; Heidelberg: Springer, 1990, pp. 171–180. doi: 10.1007/BFb0100738.
14. Shi W.J., Bi J.X. A characterization of Suzuki-Ree groups. *Sci. China, Ser. A*, 1991, vol. 34, no. 1, pp. 14–19.

¹This paper is based on the results of the 2020 Ural Workshop on Group Theory and Combinatorics.

15. Shi W.J., Bi J.X. A new characterization of the alternating groups. *Southeast Asian Bull. Math.*, 1992, vol. 16, no. 1, pp. 81–90.
16. Shi W.J. The pure quantitative characterization of finite simple groups (I). *Prog. Nat. Sci.*, 1994, vol. 4, pp. 316–326.
17. Thompson J.G. *Personal communication*. January 4, 1988.
18. Vasil'ev A.V., Grechkoseeva M.A., Mazurov V.D. Characterization of the finite simple groups by spectrum and order. *Algebra Logic*, 2009, vol. 48, no. 6, pp. 385–409. doi: 10.1007/s10469-009-9074-9.
19. Williams J.S. Prime graph components of finite groups. *J. Algebra*, 1981, vol. 69, no. 2, pp. 487–513. doi: 10.1016/0021-8693(81)90218-0.
20. Xu M.C., Shi W.J. Pure quantitative characterization of finite simple groups ${}^2D_n(q)$ and $D_l(q)$ (l odd). *Algebra Colloq.*, 2003, vol. 10, no. 3, pp. 427–443.
21. Zarrin M. A counterexample to Herzog's conjecture on the number of involutions. *Arch. Math.*, 2018, vol. 111, no. 4, pp. 349–351. doi: 10.1007/s00013-018-1195-8.
22. Zavarnitsine A.V. Finite simple groups with narrow prime spectrum. *Sib. Elektron. Mat. Izv.*, 2009, vol. 6, pp. 1–12.

Received November 14, 2020

Revised February 28, 2021

Accepted April 5, 2021

Funding Agency: The project was partially supported by the Science and Technology Research Program of Chongqing Municipal Education Commission (KJZD-K202001303) and sponsored by Natural Science Foundation of Chongqing, China (cstc2021jcyj-msxmX0511).

Jinbao Li, Department of Mathematics, Chongqing University of Arts and Sciences, Chongqing 402160, P. R. China, e-mail: leejinbao25@163.com.

Wujie Shi (Corresponding author), Department of Mathematics, Chongqing University of Arts and Sciences, Chongqing 402160, P. R. China; School of Mathematics, Suzhou University, Suzhou 215006, P. R. China, shiwujie@outlook.com.

Cite this article as: Jinbao Li and Wujie Shi. On Some Conjectures Related to Quantitative Characterizations of Finite Nonabelian Simple Groups, *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2021, vol. 27, no. 4, pp. 269–275.