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## RECOGNITION OF THE GROUP $E_6(2)$ BY GRUENBERG–KEGEL GRAPH

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The Gruenberg–Kegel graph (or the prime graph) of a finite group  $G$  is a simple graph  $\Gamma(G)$  whose vertices are the prime divisors of the order of  $G$ , and two distinct vertices  $p$  and  $q$  are adjacent in  $\Gamma(G)$  if and only if  $G$  contains an element of order  $pq$ . A finite group is called recognizable by Gruenberg–Kegel graph if it is uniquely determined up to isomorphism in the class of finite groups by its Gruenberg–Kegel graph. In this paper, we prove that the finite simple exceptional group of Lie type  $E_6(2)$  is recognizable by its Gruenberg–Kegel graph.

Keywords: finite group, simple group, exceptional group of Lie type, Gruenberg–Kegel graph (prime graph).

### REFERENCES

1. Conway J.H., Curtis R.T., Norton S.P., Parker R.A., Wilson R.A. *Atlas of finite groups*. Oxford: Clarendon Press, 1985, 252 p.
2. Geroni G.C. Note sur la resolution en nombres entiers et positifs del'equation  $x^m = y^n + 1$ . *Nouv. Ann. Math.* (2), 1870, vol. 9, pp. 469–471.
3. Gorenstein D. *Finite groups*. New York: Harper and Row, 1968, 642 p.
4. Gorenstein D., Lyons R., Solomon R. *The classification of the finite simple groups. Number 3. Part I*. Math. Surveys Monogr., vol. 40, no. 3, Providence, RI: Amer. Math. Soc., 1998, 420 p.
5. Iiyori N., Yamaki H. Prime graph components of the simple groups of Lie type over the fields of even characteristic. *J. Algebra*, 1993, vol. 155, no. 2, pp. 335–343, doi: 10.1006/jabr.1993.1048; corrigenda; *J. Algebra*, 1996, vol. 181, no. 2, pp. 659.
6. Jansen C., Lux K., Parker R., Wilson R. *An atlas of Brauer characters*. Oxford: Clarendon Press, 1995. 327 p.
7. Kondrat'ev A.S. Prime graph components of finite simple groups. *Math. USSR Sb.*, 1990, vol. 67, no. 1, pp. 235–247. doi: 10.1070/SM1990v067n01ABEH001363.
8. Kondrat'ev A.S., Mazurov V.D. Recognition of alternating groups of prime degree from the orders of their elements. *Siberian. Math. J.*, 2000, vol. 41, no. 2, pp. 294–302. doi: 10.1007/BF02674599.
9. Mazurov V.D. The set of orders of elements in a finite group. *Algebra and Logic*, 1994, vol. 33, no. 1, pp. 49–55. doi: 10.1007/BF00739417.
10. Vasil'ev A.V., Gorshkov I.B. On recognition of finite simple groups with connected prime graph. *Siberian Math. J.*, 2009, vol. 50, no. 2, pp. 233–238. doi: 10.1007/s11202-009-0027-2.
11. Vasil'ev A.V., Vdovin E.P. An adjacency criterion for the prime graph of a finite simple group. *Algebra and Logic*, 2005, vol. 44, no. 6, pp. 381–406. doi: 10.1007/s10469-005-0037-5.
12. Vasil'ev A.V., Vdovin E.P. Cocliques of maximal size in the prime graph of a finite simple group. *Algebra and Logic*, 2011, vol. 50, no. 4, pp. 291–322. doi: 10.1007/s10469-011-9143-8.
13. Williams J. S. Prime graph components of finite groups. *J. Algebra*, 1981, vol. 69, no. 2, pp. 487–513. doi: 10.1016/0021-8693(81)90218-0.
14. Zavarnitsine A.V. Finite groups with a five-component prime graph. *Siberian Math. J.*, 2013, vol. 54, no. 1, pp. 40–46. doi: 10.1134/S0037446613010060.
15. Zsigmondy K. Zur Theorie der Potenzreste. *Monatsh. Math. Phys.*, 1892, vol. 3, no. 1, pp. 265–284. doi: 10.1007/BF01692444.

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