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DIVERGENT SERIES IN THE FOURIER METHOD FOR THE WAVE EQUATION

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Series of formal solutions of two mixed problems for the wave equation are studied by a method based on the application of divergent series in the sense of Euler. The validity of this method is proved. The method is very economical in the use of well-known mathematical facts, which opens up the prospect of significant progress in the study of boundary value problems for partial differential equations.

Keywords: Fourier method, mixed problem, wave equation, divergent series, resolvent.

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