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APPLICATION OF SECOND-ORDER OPTIMIZATION METHODS FOR SOLVING AN INVERSE COEFFICIENT PROBLEM IN THE THREE-DIMENSIONAL STATEMENT

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An inverse problem of finding a temperature-dependent thermal conductivity coefficient of a substance is considered. The analysis is based on the first boundary value problem for the three-dimensional nonstationary heat equation. The sample of the substance under investigation has the form of a rectangular parallelepiped. The inverse coefficient problem is reduced to a variational problem. The root-mean-square deviation of the calculated heat flux on the surface of the body from the experimentally obtained flux is chosen as the cost functional. The paper investigates the possibility of solving the variational problem by optimization methods of the second order of convergence. On the example of a number of nonlinear problems whose coefficients are temperature-dependent, a comparative analysis of the solution of these problems by means of the gradient method and the Levenberg–Marquardt method is carried out. The accuracy of calculating the elements of the Jacobi-type matrix required to implement the Levenberg–Marquardt method has a significant impact on the convergence of the iterative process. It is essential that in our approach the elements of the Jacobi-type matrix are calculated with machine precision due to the use of the fast automatic differentiation technique. Much attention is paid to the features of solving the inverse problem associated with its three-dimensional spatial nature.

Keywords: inverse coefficient problems, nonlinear problems, three-dimensional heat equation, optimal control, numerical optimization methods, alternating direction schemes.

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