TEST POLYNOMIAL VOLTERRA EQUATION OF THE FIRST KIND IN THE PROBLEM OF INPUT SIGNAL IDENTIFICATION

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The paper discusses Volterra polynomial integral equations of the first kind that arise in describing a nonlinear dynamic system of the "input-output" type in the form of a finite segment (polynomial) of the Volterra integro-power series. A brief review of research results for such equations is given for the case when the input $x(t)$ is a scalar function of time. The most important feature of these equations is the locality (in the sense of the smallness of the right endpoint of the interval $[0, T]$) of the solution in $C_{[0,T]}$. We consider problem statements developed or outlined in the works of A. S. Apartsyn. The research part of the paper is devoted to the situation with a vector input $x(t) = (x_1(t), x_2(t))^T$. In order to study polynomial equations, we consider a test Volterra equation of the first kind. Statements are proved that determine the form of Volterra kernels guaranteeing the validity of estimates in the passage to special majorant integral equations. An algorithm for solving an equivalent Cauchy problem is presented. Unimprovable estimates expressed in terms of the Lambert function are obtained for solutions of special classes of nonlinear integral inequalities.

Keywords: nonlinear dynamic system, polynomial Volterra equations, Cauchy problem, Lambert function.

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