TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 27 No. 4 **2021**

MSC: 47N05, 37N25, 37N40

DOI: 10.21538/0134-4889-2021-27-4-125-141

THE QUASISOLUTION METHOD IN THE ANALYSIS OF CONVEX PROGRAMS WITH SINGULARITIES

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The paper is devoted to the analysis of some convex programs that are "degenerate" (improper, having no solutions in the usual sense). We propose an approach to the correction of such problems based on the ideas of the quasisolution method, which is standard in the theory of ill-posed extremal problems. The constraints of the original problem are aggregated with the use of a certain penalty function, which is explicitly included in the scheme of the quasisolution method. Two most popular variants are used: an exact penalty function and a quadratic penalty function. For each of these variants, the questions of solvability of the arising problems are studied and estimates for the convergence rate of the proposed procedures are established in the case where the input information about the problem to be analyzed is given approximately.

Keywords: convex programming, improper problem, optimal correction, quasisolution method, penalty function methods.

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Received May 12, 2021
Revised June 10, 2021
Accepted June 21, 2021

Funding Agency: This work was supported by the Russian Foundation for Basic Research (project no. 19-07-01243).

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Cite this article as: V. D. Skarin. The quasisolution method in the analysis of convex programs with singularities, *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2021, vol. 27, no. 4, pp. 125–141 .