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ON THE QUESTION OF APPROXIMATION OF SMOOTH FUNCTIONS WITH BOUNDARY LAYER COMPONENTS

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Error estimates are obtained for the method of approximation of smooth functions having boundary layer components on an interval. The method uses linear combinations of special functions obtained from the Fourier series by changes of variables. Three kinds of such variable changes are analyzed. Jackson's theorem and Kolmogorov's relations are used as underlying results. Consequently, norms of the derivative of the function being approximated appear in the estimates. The developed method enables one to significantly reduce the order of the derivative and the value of the coefficient at it in these estimates in comparison with the estimates of the error of the best polynomial approximation. Due to this, the rate of decay of the error for new approximations is significantly higher than that of polynomial ones. Expressions for the coefficients at the norms of derivatives are obtained. Analysis of the asymptotics of the remainder terms is given. A good agreement can be observed between the theoretical results and the experimental data published earlier.

Keywords: boundary layer, Fourier series, approximation without saturation, non-polynomial approximation, change of variables, error estimates, high order of convergence.

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