FRACTIONAL MCKEAN–VLASOV AND HJB–ISAACS EQUATIONS

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We study a class of abstract nonlinear fractional pseudo-differential equations in Banach spaces that includes both the McKean–Vlasov type equations describing nonlinear Markov processes and the Hamilton–Jacobi–Bellman–Isaacs equations of stochastic control and games. This approach allows us to develop a unified analysis of these equations. We obtain the well-posedness results for these equations in the sense of classical solutions, and their continuous dependence on the initial data is proved. The obtained results are extended to the case of generalized fractional equations.

Keywords: fractional McKean–Vlasov type equations, fractional HJB-Isaacs equations, mild solutions, classical solutions, Caputo–Djrbashian fractional derivative, generalized fractional derivatives.

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