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## COOPERATIVE DIFFERENTIAL GAMES WITH PARTNER SETS ON NETWORKS

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In the paper, the differential games on networks with partner sets are considered. The payoffs of a given player depend on his actions and the actions of the players from his partner set. The cooperative version of the game is proposed, and the special type of characteristic function is introduced. It is proved the constructed cooperative game is convex. Using the properties of the payoff functions and the constructed characteristic function, the Shapley Value and  $\tau$ -value are computed. It is also proved that in this special class of differential games the Shapley value is time-consistent.

Keywords: Shapley value, differential network game, time consistency, partner sets.

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